

**Computational methods in
operations research**
07 March, 2019

Exercise 1 (Knapsack problem and its generalizations). We are given 16 objects having sizes 1 (4 pcs), 2 (4 pcs), 3 (3 pcs), 6 (3 pcs) and 10 (2 pcs). We are also give a few boxes of sizes 3, 5, 7 and 12 (4 pcs from all of them). Write up IP models for the following problems, and solve them with the help of GUSEK.

- (a) Every object has a value depending on its size: $1 \rightarrow 3$, $2 \rightarrow 7$, $3 \rightarrow 11$, $6 \rightarrow 23$, $10 \rightarrow 36$. Determine the optimal packing of the given box size (that is, for each possible size, find an optimal packing of a single box of that size).
- (b) Each box has a cost depending on its size: $3 \rightarrow 4$, $5 \rightarrow 6$, $7 \rightarrow 8$, $12 \rightarrow 12$. Find a cheapest packing of the objects.
- (c) While moving, we have a truck of capacity $Q = 20$. The objects have to be packed in boxes first, and then the boxes have to be moved by the truck. How many rounds are needed to move everything?

Exercise 2. We are given three wooden beams of sizes 17, 18 and 21. We have to cut these beams into pieces of sizes 3, 4 and 6. We need twice as many pieces of size 6 as of size 4. Furthermore, we need twice as many pieces of size 3 as of size 6. Find a solution that maximizes the total number of beams that we get.

Exercise 3. Let k be a positive integer. Consider the following problem:

$$\begin{aligned} \max & -x_{2k+2} \\ 2x_1 + \dots + 2x_{2k+1} + x_{2k+2} &= 2k + 1 \\ x &\in \{0, 1\}^{2k+2} \end{aligned}$$

- (a) Verify that the branch and bound method will consider an exponential number of subproblems.
- (b) Write up the LP relaxation of the problem, and convert it to standard form. What are the primal feasible solutions? What are the primal feasible optimal solutions?
- (c) Write up the Gomory-cuts corresponding to one of the optimal bases, and solve the extended problem. What happens?
- (d) Do a comparison using GUSEK: Write up the IP model, and check the time needed to solve the problem for different values of k . Do the same with the Gomory-cut option enabled.